Creep Bending of Rectangular Plates with Large Deflection

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1. Introduction

Two types of nonlinearities may occur in a structure. One is nonlinear stress-strain relationship. This may be caused by the presence of time-dependent creep strain and/or the time-independent plastic strain. The other type of nonlinearity is the nonlinear strain displacement relationship, which has to be considered when the strain becomes finite. It has been shown by Lin[1] that to calculate the deflection and strain distribution in a plate, the creep and plastic strains can be considered as an additional set of lateral load and edge moments. This method of equivalent load has been widely used in the analysis of inelastic bending of plates.[2–5]

The small deflection plate theory neglects the nonlinear terms of displacement derivatives in strain calculations. Considering these nonlinear terms, Von Karman has derived his well-known differential equations of large deflection of elastic thin plates. These equations have been applied to obtain large deflection solutions of elastic plates with various edge conditions.[6–9]

The use of plates to sustain high loads at elevated temperatures has been increased rapidly during the last few decades. Under such condition, both types of nonlinearities may occur. The analysis of a circular plate under axisymmetrical loadings with these nonlinearities has been shown by Naghdi.[10] The present Note aims to give a method for the solution of rectangular plates with both of these types of nonlinearities.

2. Elastic Plates with Large Deflection

By extending the equivalent force concept of inelastic problems, the nonlinear part of strain in the strain-displacement relationship is treated as an equivalent force. Then the same linear elastic problem results as the one derived from infinitesimal strain but with some additional loads represented by the nonlinear terms. The solution is obtained by using the known solution of the linear problem through an iteration procedure. When applied to problems with both material and geometrical nonlinearities, this method needs no modification.

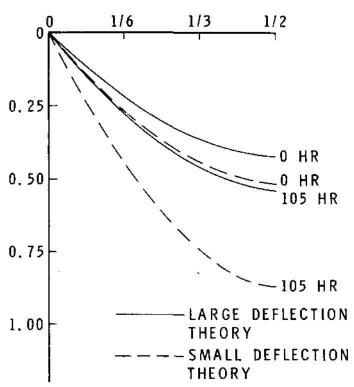

Fig. 1 Creep deflection of a square plate.

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The governing equations for elastic plates with large deflection are the well-known Von Karman equations

$$\nabla^4 w = \frac{1}{D}\left(q + \frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right) \quad (1)$$

$$\nabla^4\phi = Eh[(\partial^2 w/\partial x\partial y)^2 - (\partial^2 w/\partial x^2)(\partial^2 w/\partial y^2)] \quad (2)$$

where $\nabla^4 = \nabla^2\nabla^2 = [(\partial^2/\partial x^2) + (\partial^2/\partial y^2)][(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]$, w is the deflection of the plate, h the plate thickness, E the Young's modulus, ν the Poisson's ratio, $D = Eh^3/12(1 - \nu^2)$, q the lateral load, and ϕ a stress function defined by $N_x = (\partial^2\phi/\partial y^2)$, $N_y = (\partial^2\phi/\partial x^2)$, $N_{xy} = -(\partial^2\phi/\partial x\partial y)$ in which N_x, N_y, and N_{xy} are the in-plane forces.

Now let

$$q' = \frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} \quad (3)$$

$$F = Eh[(\partial^2 w/\partial x\partial y)^2 - (\partial^2 w/\partial x^2)(\partial^2 w/\partial y^2)] \quad (4)$$

then Eq. (1) becomes the governing equation of the bending of plates subjected to lateral load $q + q'$ and Eq. (2) that of a plane stress problem under certain body forces derivable from a potential function F. With the solutions of these linear problems known, the solution of a plate with large deflection can be obtained as follows. First a trial deflection w_1 is assumed. As a good approximation, the deflection based on the small deflection theory can be used as this trial deflection. This w_1 is used to evaluate the potential function F_1 from Eq. (4). A first approximation of the stress function ϕ_1 can be obtained as the solution of the plane stress problem due to F_1. Substituting ϕ_1 and w_1 into Eq. (3), we get a first trial equivalent lateral load q_1'. Then the solution of the plate problem gives a second deflection w_2 due to lateral load $q + q_1'$. The process is repeated to find $w_3, w_4, \ldots w_n$ until w_{n-1} and w_n coincide.

3. Creep Bending of Plates with Large Deflection

The governing equations of creep bending of plates with large deflection are given by[1]

$$\nabla^4 w = (1/D)(q + q' + \bar{q}) \quad (5)$$

$$\nabla^4\phi = F + \bar{F} \quad (6)$$

in which $\bar{q}$ is the equivalent lateral load due to creep strain

$$\bar{q} = \frac{E}{1-\nu^2}\frac{\partial^2}{\partial x^2}\int(e_x'' + \nu e_y'')zdz - 2(1-\nu)\frac{\partial^2}{\partial x\partial y} \times \int e_{xy}''zdz + \frac{E}{1-\nu^2}\frac{\partial^2}{\partial y^2}\int(e_y'' + \nu e_x'')zdz \quad (7)$$

and $\bar{F}$ the equivalent body force potential

$$\bar{F} = -E\left[\frac{\partial^2}{\partial y^2}\int e_x''dz + \frac{\partial^2}{\partial x^2}\int e_y''dz - \frac{\partial^2}{\partial x\partial y}\int 2e_{xy}''dz\right] \quad (8)$$

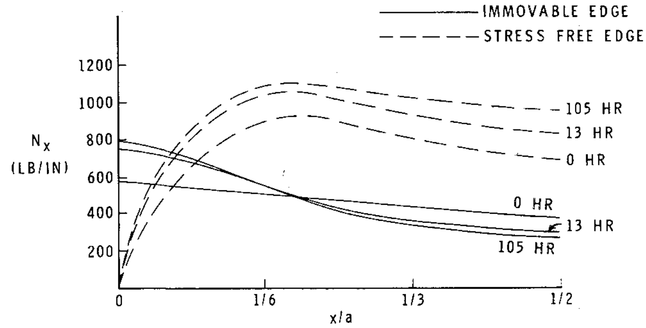

Fig. 2 N_x along center line of plate.

In Eqs. (7) and (8), e_x'', e_y'', and e_{xy}'' are the components of creep strain and the integration is over the thickness.

Consider a particular point in the plate. The stresses at this point vary continuously with time due to creep. The smooth stress time curve can be approximated by a series of finite steps, each of which consists of a constant stress period Δt followed by an instantaneous increment of stress. The increment of creep strains in the constant stress interval can be obtained as fully described in Ref. 1. The total creep strain at any time is the sum of creep strain increments of all the time steps. Hence, the creep strains and the equivalent forces readily can be calculated at the end of each time step. Our problem can then be solved in a manner similar to that used in Sec. 2.

At time $t = 0$ there is no creep strain, $\bar{q} = \bar{F} = 0$. An elastic solution is obtained first. At any subsequent time step, we use the previous deflection as the first trial deflection to start the iteration. In the iteration process, $\bar{q}$ and $\bar{F}$ remain unchanged while w,ϕ,q' and F approach their correct values.

4. Numerical Example

The procedure in Secs. 2 and 3 is applied to a 7075-T6 aluminum alloy square plate $\frac{1}{2}$ in. $\times$ 24 in. $\times$ 24 in. undergoing large creep deflection at 600°F. The lateral load q is uniform and equals 10 psi. The uniaxial creep characteristics of the material are approximated by $e_c = At^K \sinh(B\sigma)$, in which t is time in hours, and the material constants are $A = 5.25 \times 10^{-7}$, $B = 1.92 \times 10^{-3}$, $K = 0.66$, $E = 5.2 \times 10^6$, and $\nu = 0.32$. Two simply supported edge conditions are considered. One is with zero sectional forces, and the other with zero displacement. From the elastic solutions, the deflection of the plate due to a unit load at (ξ,η) is given by

$$w = \frac{a^2}{\pi^3 D} \sum_m \left(1 + m\pi \coth m\pi - \frac{m\pi y_1}{a}\coth\frac{m\pi y_1}{a} - \frac{m\pi\eta}{a}\coth\frac{m\pi\eta}{a}\right) \frac{\sinh(m\pi\eta/a)\sinh(m\pi y_1/a)\sin(m\pi\xi/a)\sin(m\pi x/a)}{m^3 \sinh m\pi} \quad (9)$$

where m denotes an integer, $y_1 = a - y$ and $y \geq \eta$. For $y < \eta$, y_1 is replaced by y and η by $a - \eta$. The plate is divided into 12 $\times$ 12 grid spacings. The influence coefficients for elastic moments are computed by finite differences giving, for instance,

$$Mx_{ij} = D[(1/\Delta x^2)(-w_{i-1,j} + 2w_{ij} - w_{i+1,j}) - (\nu/\Delta y^2)(-w_{i,j-1} + 2w_{ij} - w_{i,j+1})] \quad (10)$$

The stress influence coefficients for the plane stress problem are obtained by use of the computer program of Ref. 11 for both the stress free and immovable edges. The calculated results are shown in Figs. 1–3. The deflections for both cases are almost the same. The inplane forces vary differently.

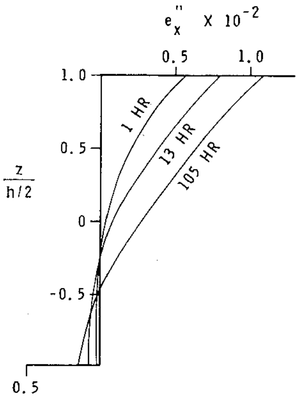

Fig. 3 Variation of creep strain at center of plate (stress free edge).

The numerical example exhibited an extremely rapid convergence. The first time increment was taken to be 0.0001 hr. The subsequent increment was doubled in each step. For a relative accuracy of 0.0002%, three cycles of iteration were enough for most steps and in no case exceeded five cycles. The total computing time took only 8 min, of which 5 min were spent in establishing the influence coefficients.

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Penetration of the Flame Front through a Fine Metal Layer in a Solid Propellant

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In spite of the existence of a great number of works dealing with the investigations of nonsteady solid combustion, a comprehensive solid combustion theory is not available which takes into account the total amount of known experimental data on a combustion mechanism. The aforementioned experimental data comprise, for example, some results on the space extension of the heat release region, on the stability conditions of a combustion process, on the combustion of solid propellant under nonsteady pressure variations, and on the conditions of the extinction and ignition. The insufficiency of existing theories is caused mainly by the difficulties in experimental investigation of the combustion process.

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